Int. J. Heat Mass Transfer. Vol. 22, pp. 657–668

0017-9310/79/0501-0657 $02.00/0

RADIATIVE HEAT TRANSFER IN HORIZONTAL MAGNETOHYDRODYNAMIC CHANNEL FLOW WITH BUOYANCY EFFECTS AND AN AXIAL TEMPERATURE GRADIENT

J. B. HELLIWELL and M. F. MOSA
School of Mathematics, University of Bradford, West Yorkshire, England

(*Received* 20 *January* 1978 *and in revised form* 10 *August* 1978)

Abstract—Studies of the flow of a hot electrically conducting fluid in a rectangular horizontal channel with transverse magnetic field and significant heat transfer by thermal radiation are extended to take particular account of two effects. In the first of these a power law dependence of the absorption coefficient upon the temperature is taken into consideration. Comparison with the results for constant absorption coefficient shows the consequences to be not particularly marked. The second effect examined is that of the influence of buoyancy forces and convective heat transfer when the channel walls are differentially and non-uniformly heated. In this case there is a significant contribution to the field profiles which are considerably distorted from those when the wall temperatures are uniform.

Throughout the investigations, the gas is taken to have general opacity for radiative transfer and the walls are of arbitrary electrical conductivity and emissivity. Molecular heat conduction, viscosity and ohmic dissipation are all taken into account. A few exact solutions are obtained but in general the governing differential equations are integrated numerically and the results presented graphically.

NOMENCLATURE

b, dimensionless magnetic field;
c, velocity of light;
c_p, c_v, specific heats;
d, wall thickness;
g, gravitational acceleration;
h, channel semi-width;
k, coefficient of thermal conductivity;
m, n, indices [see equation (3)];
p, pressure;
q, radiative flux;
u, velocity;
v, dimensionless velocity;
x, y, z, coordinates;
Bo, Boltzmann number;
B_0, applied magnetic field;
B_x, magnetic field component;
E, electric field number;
E_0, electric field;
Ec, Eckert number;
F, defined $PrEc$;
Gr, Grashof number;
I, current number;
J, current density;
K, wall conductivity number;
K_y, absorption coefficient parameter [see equation (3)];
M, Hartmann number;
N, defined Re/Bo;
Pr, Prandtl number;
Q, dimensionless radiative flux;
Re, Reynolds number;
Re_m, magnetic Reynolds number;
$T^*, T, \bar{T}$, temperatures [see equation (1)];
U, mean fluid velocity.

Greek symbols

α, absorption coefficient;
β, coefficient of thermal expansion;
γ, specific heat ratio;
ε, wall emissivity;
ρ, density;
η, dimensionless y coordinate;
σ, electrical conductivity;
$\tilde{\sigma}$, Stefan's constant;
$\theta, \bar{\theta}, \Theta$, dimensionless temperatures [see equation (28)];
τ, scale factor [see equation (1)];
μ, coefficient of viscosity;
μ_e, permeability;
ω, Bouguer number;
Σ, dimensionless radiative energy density;
$\bar{\Sigma}$, radiative energy density.

Subscripts

1, value at lower wall;
2, value at upper wall;
x, y, z, vector components.

Superscripts

1, part of variable independent of x [see equations (13)–(15)];
2, part of variable dependent on x [see equations (13)–(15)].

1. INTRODUCTION

SIMPLE Hartmann flow of an electrically conducting incompressible fluid in a rectangular channel with downstream pressure gradient and transverse applied magnetic field is a well understood basic configuration. However, a considerable number of physical attributes of a realistic fluid which could well be significant are omitted from this model. Among these are effects of thermal conduction and convection due to buoyancy. In addition, when the temperature is high, the importance of radiative heat transfer is quite overlooked.

In recent years a number of studies have been carried out specifically concerned with elucidating the influence of the various mechanisms of heat transfer in channel flow of an electrically conducting fluid in the presence of a magnetic field. These investigations have application in problems associated with the cooling of nuclear reactors as well as in more wide ranging areas involving the use of magnetohydrodynamic pumps and generators. Additionally they also serve to enhance our broad understanding of motions involving plasmas and conducting fluids generally.

The fundamental effects of convective heat transfer in magnetohydrodynamics have been analysed by Siegel [1] and Alpher [2], among others. For a vertical channel with constant wall temperatures Gershuni and Zhukhovitsky [3] examined the problem whilst Yu [4] considered the same configuration but with walls of linearly varying temperature. Particular emphasis was placed upon the effect of electrical conductivity of the walls by Chang and Yen [5]. The influence of buoyancy forces in a horizontal channel with non-conducting walls has been studied by Gill and Casal [6] and Gupta [7]. More recently Jana [8] has presented an exact solution for the free and forced convective flow between two horizontal finitely conducting walls with linear temperature variation. The effects of convection have also been investigated by Soundalgekar [9].

All the above analyses however take no account of any possible effects of radiative heat transfer. Some study of the interaction in the absence of electromagnetic effects has been made by Greif, Habib and Lin [10] and Viskanta [11] has examined some aspects of the influence of radiation and magnetic field on the flow in a horizontal channel. Gupta and Gupta [12] have discussed the effect of radiation on the convection in an electrically conducting fluid in a vertical channel. However in all these investigations, in order to simplify the analyses and generate exact solutions, the optically thin limit has been employed for radiative transfer with consequent loss of generality. An earlier paper by one of the present authors, Helliwell [13] has examined the effect of radiation upon simple Hartmann flow under conditions of general opacity using the so-called differential approximation for radiative transfer and, in a sequel, Helliwell [14] has gone on to introduce the further effect of thermal conduction. The purpose of the present paper is to extend this previous study of the transverse variation of the flow profiles so as to take account of the dependence of the absorption coefficient upon the temperature and density and to generalise the configuration to the case when the horizontal channel walls, having different linear axial temperature variations, are of arbitrary electrical conductivities. The effects of buoyancy are also introduced.

2. THE MODEL AND GOVERNING EQUATIONS

Consider a long channel of great width and of height $2h$ lying between two horizontal walls of arbitrary electrical conductivity. Take an origin of coordinates in some appropriate cross-section so that the walls become the planes $y = \pm h$ and the x axis lies parallel to the direction of flow of an electrically conducting fluid which fills the channel. The temperatures T^* of the walls are taken to vary linearly with x so that

$$T^* = T_1^* = T_1 + \frac{\tau x}{h}\bar{T}_1, \qquad T^* = T_2^* = T_2 + \frac{\tau x}{h}\bar{T}_2, \tag{1}$$

where T_1, T_2, $\bar{T}_1$, $\bar{T}_2$ are constants and τ is a suitable scale factor. Throughout this paper suffices (1) and (2) refer to values at the lower and upper wall respectively. The bounds of the channel normal to the z axis are assumed to be perfectly conducting electrodes set infinitely far apart so that any dependence upon the z coordinate vanishes from the equations. The electrical conductivities of the fluid and walls are all supposed constant and are denoted respectively by σ, σ_1 and σ_2. An externally applied magnetic field B_0 is applied uniformly across the channel in the direction of the y axis.

In order to ease somewhat the mathematical analysis without, it is hoped, much loss of reality the Boussinesq approximation to the equation of state is introduced, as in most previous studies. Thus the fluid is supposed incompressible so far as direct contributions from the density ρ to the conservation equations are concerned apart from that arising from the buoyancy term in the equation of momentum. We write

$$\rho = \rho_1\left[1 - \frac{\beta}{T_1}(T^* - T_1)\right], \tag{2}$$

where β is a coefficient of thermal expansion. A further fairly gross assumption is made that the coefficients of viscosity and thermal conductivity μ and k respectively are constants. However the absorption coefficient α for a grey gas at fairly high temperatures is known to be a more rapidly varying function of the temperature than either μ or k, and following Armstrong *et al.* [15] may be written in the form

$$\alpha = K_\alpha \rho^m T^{*n}. \tag{3}$$

For instance when $\rho \simeq 1.3 \times 10^{-3}$ g/cm^3 and $T^* \simeq 10^4$ K one has $m \simeq 1$, $n \simeq 5$. The variation of density with temperature is also retained in this form for α. One of the objects of the present paper is to examine the consequence of taking this variable form for α as compared with the commonly used constant coefficient.

For fully developed laminar flow the continuity equation is identically satisfied with the only non-zero velocity component horizontal and a function of y alone. This is written $u = u(y)$. Apart from cases when the gas has an extremely high temperature ($> 10^5$ K) it is known that no contribution arises from radiative effects in the equation of conservation of momentum, and only that from radiative flux in the equation of conservation of energy; see for instance Vincenti and Kruger [16]. The equation of motion is therefore unchanged from that of conventional magnetohydrodynamics, which together with the electromagnetic equations may be found in standard texts, as for example Shercliff [17]. From these it follows that the electric current density has a single component $J = J(y)$ in a direction parallel to the z axis. A component $B_x = B_x(y)$ of magnetic field is induced parallel to the flow whilst the transverse component remains parallel to the y axis and has constant magnitude B_0. Furthermore the electric field possesses a single constant component E_0 parallel to the z axis. This set of equations thus reduces to

$$\frac{\partial p}{\partial x} = \mu \frac{d^2 u}{dy^2} - B_0 J, \qquad (4)$$

$$\frac{\partial p}{\partial y} = J B_x - \rho g, \qquad (5)$$

$$J = \frac{-1}{\mu_e} \frac{dB_x}{dy} = \sigma(E_0 + u B_0), \qquad (6)$$

where p is the pressure, μ_e is the permeability and g is the gravitational acceleration. Elimination of p and J from these equations following the use of equation (2) leads to the relationship

$$\frac{B_0}{\mu_e} \frac{d^2 B_x}{dy^2} + \mu \frac{d^3 u}{dy^3} = \rho_1 g \frac{\beta}{T_1} \frac{\partial T^*}{\partial x}. \qquad (7)$$

Hence, since the left hand side is a function of y alone, it follows that the most general possible form for T^* is linear in x, consistent with the form (1) for the temperatures of the walls. We write

$$T^* = T(y) + \frac{\tau x}{h} \bar{T}(y). \qquad (8)$$

Taking account of the above forms and introducing the contributions from the radiative flux $\mathbf{q}$ (of which the component in the z direction is identically zero, the others q_x, q_y being functions of both x and y), the equation of energy may be written

$$k\left(\frac{d^2 T}{dy^2} + \frac{\tau x}{h} \frac{d^2 \bar{T}}{dy^2}\right) - \rho_1 \frac{c_v u}{h} \tau \bar{T} - \frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} + \frac{J^2}{\sigma} + \mu\left(\frac{du}{dy}\right)^2 = 0 \qquad (9)$$

where c_v is the specific heat at constant volume.

To close the system of equations it is necessary to introduce the equations of radiative heat transfer. In its exact form it is well known that this gives rise to a coupled system of integro-differential relationships, and to circumvent the consequent formidable analytical difficulties various approximations have been introduced. Here we employ the so-called differential approximation, see Vincenti and Kruger [16], which replaces the exact equations by a system of approximating differential equations. For a two-dimensional configuration these may be written

$$\alpha c \bar{\Sigma} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} = 4\tilde{\sigma}\alpha T^{*4}, \qquad (10)$$

$$c \frac{\partial \bar{\Sigma}}{\partial x} + 3\alpha q_x = 0, \qquad (11)$$

$$c \frac{\partial \bar{\Sigma}}{\partial y} + 3\alpha q_y = 0, \qquad (12)$$

where c is the velocity of light, $\tilde{\sigma}$ is Stefan's constant and $\bar{\Sigma}$ is the radiative energy density.

Equations (8)–(12) indicate quite clearly that strictly the model is two-dimensional and the equations partial differential. Recall though that we are concerned with two particular problems which may be studied separately and indeed are so analysed later.

For the first problem interest centres on the effect of a variable absorption coefficient on the predicted character of the flow in a channel with walls of uniform temperature. For this the analysis is straightforward. In all equations the parameter τ is set identically zero and a solution exists in which all variables are functions of y alone, with $q_x \equiv 0$, so that the equations degenerate into an ordinary differential system.

In the second problem the effects of the non-uniformity of the wall temperature are taken into account but, as will be seen later, the absorption coefficient may be assumed constant. In this case it is more difficult to justify progress without a study of the full partial differential system. The model, even with constant absorption coefficient, is not capable of development using a regular power series expansion in x. A basic difficulty lies in the non-linearity in x of the exact form of the RHS of equation (10). Were this not the case a solution could be found such that all variables were at most exactly linear in x. Thus the fairly crude approximation is made, assuming a constant absorption coefficient, that the RHS of equation (10) may be replaced by its linear approximation in x, viz

$$4\tilde{\sigma}\alpha T^4 + 16\tilde{\sigma}\alpha \frac{\tau}{h} T^3 \bar{T} x. \qquad (13)$$

Whilst the solution which then follows cannot be interpreted as a first approximation to a power series expansion in x for the solution to the problem in the large, it may nevertheless be regarded in the sense indicated above as a linear approximation to the solution. It can therefore serve to provide an indication of the effect of non-uniformity of wall temperature upon the transverse variation of the profiles of velocity, temperature and radiative flux at any particular cross-section of the channel.

The mathematical analysis for both the above problems may usefully be combined in a single formulation. Thus set

$$c\bar{\Sigma} = \bar{\Sigma}^{(1)} + \frac{\tau x}{h}\bar{\Sigma}^{(2)}, \tag{14}$$

$$q_x = \tau q_x^{(1)} + \frac{\tau^2 x}{h} q_x^{(2)}, \quad q_y = q_y^{(1)} + \frac{\tau x}{h} q_y^{(2)}. \tag{15}$$

The absorption coefficient, using equations (2), (3) and (8) with $\tau = 0$, becomes

$$\alpha = \frac{K_r \rho_1^m}{T_1^m}[T_1 - \beta(T - T_1)]^m T^n, \tag{16}$$

with m and n both set zero for the second problem.

The equations of energy and radiative transfer, on identifying the terms independent of, and linear in, x become

$$k\frac{d^2 T}{dy^2} - \rho_1 \frac{c_v u}{h}\tau T - \frac{\tau^2}{h} q_x^{(2)} - \frac{dq_y^{(1)}}{dy} + \frac{J^2}{\sigma} + \mu\left(\frac{du}{dy}\right)^2 = 0, \tag{17}$$

$$k\frac{d^2 \bar{T}}{dy^2} - \frac{dq_y^{(2)}}{dy} = 0, \tag{18}$$

$$\alpha\bar{\Sigma}^{(1)} + \frac{\tau^2}{h} q_x^{(2)} + \frac{dq_y^{(1)}}{dy} - 4\bar{\sigma}\alpha T^4 = 0, \tag{19}$$

$$\frac{d\bar{\Sigma}^{(1)}}{dy} + 3\alpha q_y^{(1)} = 0, \tag{20}$$

$$\alpha\bar{\Sigma}^{(2)} + \frac{dq_y^{(2)}}{dy} - 16\bar{\sigma}\alpha T^3 \bar{T} = 0, \tag{21}$$

$$\frac{d\bar{\Sigma}^{(2)}}{dy} + 3\alpha q_y^{(2)} = 0, \tag{22}$$

$$\bar{\Sigma}^{(2)} + 3\alpha h q_x^{(1)} = 0, \tag{23}$$

$$\alpha q_x^{(2)} = 0. \tag{24}$$

It remains to state the boundary conditions at the walls. Continuity of velocity and temperature require

$$\begin{aligned} &u = 0,\ T = T_1,\ \bar{T} = \bar{T}_1 \text{ at } y = -h; \\ &u = 0,\ T = T_2,\ \bar{T} = \bar{T}_2 \text{ at } y = +h. \end{aligned} \tag{25}$$

The electromagnetic conditions may be expressed entirely in terms of B_x. Following Shercliff [17] if d_1 and d_2 denote the wall thickness (both supposed much less than the channel height) it follows that

$$\begin{aligned} &\frac{dB_x}{dy} - \frac{\sigma B_x}{\sigma_1 d_1} = 0 \text{ at } y = -h, \\ &\frac{dB_x}{dy} + \frac{\sigma B_x}{\sigma_2 d_2} = 0 \text{ at } y = +h. \end{aligned} \tag{26}$$

Finally for the case of non-black walls Cess [18] has derived the form for the radiative conditions appropriate to the differential approximation. Thus if the upper and lower walls have emissivities ε_2 and ε_1 respectively, then

$$\begin{aligned} &\bar{\Sigma}^{(1)} + \left(\frac{4}{\varepsilon_1} - 2\right) q_y^{(1)} = 4\bar{\sigma} T_1^4, \\ &\bar{\Sigma}^{(2)} + \left(\frac{4}{\varepsilon_1} - 2\right) q_y^{(2)} = 16\bar{\sigma} T_1^3 \bar{T}_1 \text{ at } y = -h; \\ &\bar{\Sigma}^{(1)} - \left(\frac{4}{\varepsilon_2} - 2\right) q_y^{(1)} = 4\bar{\sigma} T_2^4, \\ &\bar{\Sigma}^{(2)} - \left(\frac{4}{\varepsilon_2} - 2\right) q_y^{(2)} = 16\bar{\sigma} T_2^3 \bar{T}_2 \text{ at } y = +h. \end{aligned} \tag{27}$$

3. PARAMETERS AND DIMENSIONLESS EQUATIONS

A considerable number of named dimensionless parameters occur in this model and in order to portray these explicitly it is now necessary to introduce a set of dimensionless variables. Take the mean fluid velocity as reference velocity $\mathbf{U}$, the temperature of the lower wall at $x = 0$ as reference temperature T_1, the emissive power $\bar{\sigma} T_1^4$ of a black wall at the reference temperature as base for the radiative variables and introduce the semi-channel height as a dimension of length. Thus set

$$\begin{aligned} &u = Uv, && T = T_1\theta, && \bar{T} = T_1\bar{\theta}, \\ &\mathbf{q} = \bar{\sigma} T_1^4 \mathbf{Q}, && \bar{\Sigma} = \bar{\sigma} T_1^4 \Sigma, && y = h\eta, \\ &B_x = B_0 \mu_e \sigma U h b, && T_2 = T_1\Theta, && \bar{T}_1 = T_1\Theta_1, \\ &\bar{T}_2 = T_1\Theta_2. \end{aligned} \tag{28}$$

Define

$$\begin{aligned} &M = B_0 h\left(\frac{\sigma}{\mu}\right)^{1/2}, \quad Re = \frac{h\rho_1 U}{\mu}, \quad Ec = \frac{U^2}{c_p T_1}, \\ &\gamma = \frac{c_p}{c_v}, \quad Re_m = \mu_e \sigma U h, \quad Pr = \frac{\mu c_p}{k}, \\ &Gr = \frac{g\rho_1 \beta h^2}{\mu U}, \quad Bo = \frac{\rho_1 U^3}{\bar{\sigma} T_1^4}, \quad \omega = K_r \rho_1^m T_1^n h = \alpha_0 h. \end{aligned} \tag{29}$$

Introduce also

$$\begin{aligned} &K_1 = \frac{\sigma_1 d_1}{\sigma h}, \quad K_2 = \frac{\sigma_2 d_2}{\sigma h}, \\ &E = \frac{E_0}{UB_0}, \quad I = \frac{J_0 h \mu_e}{B_0 Re_m}, \end{aligned} \tag{30}$$

where J_0 is the mean current density.

It is also useful to write

$$N = Re/Bo, \quad F = (Pr)(Ec). \tag{31}$$

Equations (6), (7) and (17)–(24) now become in dimensionless form

$$\frac{db}{d\eta} + v + E = 0 \tag{32}$$

$$\frac{d^3 v}{d\eta^3} + M^2 \frac{d^2 b}{d\eta^2} - Gr\tau\bar{\theta} = 0 \tag{33}$$

$$\frac{d^2\theta}{d\eta^2}+F\left[\left(\frac{dv}{d\eta}\right)^2+M^2\left(\frac{db}{d\eta}\right)^2 -N\left(\frac{dQ_y^{(1)}}{d\eta}+\tau^2 Q_x^{(2)}\right)-\frac{Re\tau}{\gamma Ec}v\bar{\theta}\right]=0 \quad (34)$$

$$\frac{d^2\bar{\theta}}{d\eta^2}-FN\frac{dQ_y^{(2)}}{d\eta}=0 \quad (35)$$

$$\frac{dQ_y^{(1)}}{d\eta}+\omega\{\theta^n[1-\beta(\theta-1)]^m\}\Sigma^{(1)} +\tau^2 Q_x^{(2)}-4\omega\theta^{n+4}[1-\beta(\theta-1)]^m=0 \quad (36)$$

$$\frac{d\Sigma^{(1)}}{d\eta}+3\omega\{\theta^n[1-\beta(\theta-1)]^m\}Q_y^{(1)}=0 \quad (37)$$

$$\frac{dQ_y^{(2)}}{d\eta}+\omega\{\theta^n[1-\beta(\theta-1)]^m\}\Sigma^{(2)} -16\omega[1-\beta(\theta-1)]^m\theta^{n+3}\bar{\theta}=0 \quad (38)$$

$$\frac{d\Sigma^{(2)}}{d\eta}+3\omega\{\theta^n[1-\beta(\theta-1)]^m\}Q_y^{(2)}=0 \quad (39)$$

$$\Sigma^{(2)}+3\omega\{\theta^n[1-\beta(\theta-1)]^m\}Q_x^{(1)}=0 \quad (40)$$

$$Q_x^{(2)}=0. \quad (41)$$

Using equation (6) to rearrange the electromagnetic conditions, the full set of boundary conditions (25), (26) and (27) becomes

$$v=0,\ \theta=1,\ \bar{\theta}=\Theta_1,\ b=-K_1E,$$
$$\Sigma^{(1)}+\left(\frac{4}{\varepsilon_1}-2\right)Q_y^{(1)}=4, \quad (42)$$
$$\Sigma^{(2)}+\left(\frac{4}{\varepsilon_1}-2\right)Q_y^{(2)}=16\Theta_1,\ \text{at}\ \eta=-1.$$

$$v=0,\ \theta=\Theta,\ \bar{\theta}=\Theta_2,\ b=K_2E,$$
$$\Sigma^{(1)}-\left(\frac{4}{\varepsilon_2}-2\right)Q_y^{(1)}=4\Theta^4, \quad (43)$$
$$\Sigma^{(2)}-\left(\frac{4}{\varepsilon_2}-2\right)Q_y^{(2)}=16\Theta^3\Theta_2,\ \text{at}\ \eta=1.$$

It should be noted that the parameters E, I, K_1, K_2 are not independent. Integration of equation (32) across the channel and application of the boundary conditions yields the relationship

$$I=E+1=\frac{K_1+K_2}{K_1+K_2+2}. \quad (44)$$

4. THE EFFECT OF VARIABLE ABSORPTION COEFFICIENT

In studies previously made in this field the variation of the absorption coefficient has been neglected. Thus we now examine the influence of this variation upon the distribution of temperature and radiative flux in the channel. In order to avoid any confusion the case of a channel with walls at uniform, but possibly different, temperatures is considered and then a direct comparison may be made with the earlier work of Helliwell [14] for constant coefficient. Thus in the system of equations (32)–(44) set $\tau\equiv 0$. All variables carrying superscript (2) as well as $\bar{\theta}$ become zero and the equations for v and b separate from the remainder to give

$$v=M\left\{\frac{\cosh M-\cosh M\eta}{\sinh M-M\cosh M}\right\} \quad (45)$$

$$b=\frac{M\eta\cosh M-\sinh M\eta}{\sinh M-M\cosh M}+\frac{2\eta+K_1-K_2}{2+K_1+K_2}. \quad (46)$$

These forms are then substituted into equations (34), (36) and (37) the solutions of which are to be determined under boundary conditions (42) and (43).

The solution of this problem is first obtained for three special cases with constant absorption coefficient. In circumstances when the temperature difference between the walls is small the equations may be linearised with respect to the temperature perturbation from that of the lower wall. An analytical form for the solution may be obtained but is not given here owing to its algebraic complexity; details can be found in Mosa [19]. Secondly the solution is established in the optically thick limit for $\omega\gg 1$. Then

$$Q_y^{(1)}=-(16/3\omega)\theta^3\,d\theta/d\eta.$$

Employing a transformation due to Ozisik [20] one finds that the energy equation may be integrated so that the temperature is given by the solution of the algebraic equation

$$\theta+\frac{4NF}{3\omega}\theta^4=c_1+c_2\eta -F(c_3\eta^2-c_4\cosh M\eta+c_5\cosh 2M\eta), \quad (47)$$

where

$$c_1=\frac{2NF}{3\omega}(\Theta^4+1)+F(c_3-c_4\cosh M+c_5\cosh 2M),$$

$$c_2=\tfrac{1}{2}(\Theta-1)+\frac{2NF}{3\omega}(\Theta^4-1),$$

$$c_3=\tfrac{1}{2}M^2c_6^2,$$

$$c_4=\frac{2Mc_6}{\sinh M-M\cosh M},$$

$$c_5=\frac{M^2}{(\sinh M-M\cosh M)^2},$$

$$c_6=\frac{M\cosh M}{\sinh M-M\cosh M}+\frac{2}{K_1+K_2+2}.$$

In the third special case the problem is analysed under the thin limit when $\omega\ll 1$. The appropriate radiative flux–temperature relationship is then

$$\frac{dQ_y^{(1)}}{d\eta}=4\omega\theta^4$$

and insertion of this into the energy equation (34) leaves a non-linear differential equation which in general cannot be integrated. However if the first special case for walls of similar temperature is now combined with the present limit the integration may be performed and one finds

$$\theta = F\left\{\frac{4M^2c_5}{S(S^2-4M^2)}(2M\sinh 2M\eta\sinh 2S\eta - S\cosh 2M\eta\cosh 2S\eta) - \frac{M^2c_4}{S(S^2-M^2)}(2M\cosh S\eta - S\cosh 2S\eta\cosh M\eta) - \frac{2c_3}{S^2}\cosh 2S\eta\right\} - \tfrac{3}{4}\cosh 2S\eta + c_7\frac{\cosh S\eta}{\cosh S} + c_8\frac{\sinh S\eta}{\sinh S}, \quad (48)$$

where

$$S = (16\omega NF)^{1/2},$$

$$c_7 = F\left[\frac{4M^2c_5}{S(S^2-4M^2)}(S\cosh 2M\cosh 2S - 2M\sinh 2M\sinh 2S) - \frac{M^2c_4}{S(S^2-M^2)}(S\cosh 2S\cosh M - 2M\cosh S) + \frac{2c_3}{S^2}\cosh 2S\right] + \tfrac{3}{4}\cosh 2S + \tfrac{1}{2}(\Theta+1),$$

$$c_8 = \tfrac{1}{2}(\Theta-1).$$

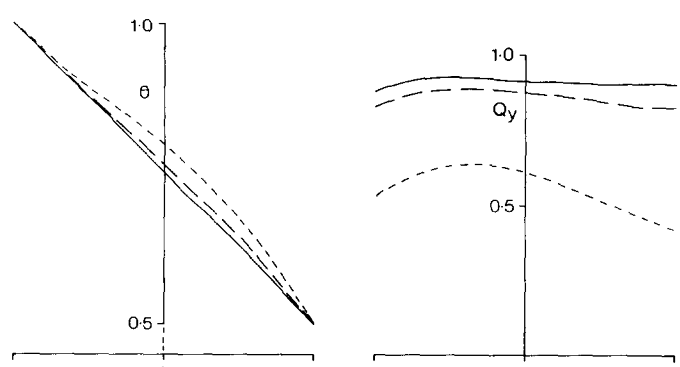
In cases with arbitrary values of the absorption coefficient and wall temperature ratio recourse must be had to digital computation. The appropriate forms of equations (34), (36) and (37) may be written as a system of four first order differential equations with mixed boundary conditions. Iterative computer routines are available for the solution of such systems, see, for instance, NAG [21]. The iteration may be started from one of the foregoing exact solutions.

When the results of calculation are compared with those derived from the foregoing special cases, a much closer agreement is apparent between the exact and approximate solutions in the cases of the linearised formulation and optically thin limit than under the optically thick limit. Thus conclusions drawn from analyses based upon the latter should be treated with greater circumspection. Profiles for various values of the parameters are presented in Figs. 1–5. The results confirm those previously obtained less satisfactorily by Helliwell [13, 14] using methods of analogue computation.

Turning now to the case with variable absorption coefficient, which again is not amenable to exact mathematical solution, the distributions are obtained by digital computation in a similar manner to that described above. A selection is presented graphically in Figs. 1–5. The iteration in this case however is started from the associated, previously computed, solution with constant coefficient.

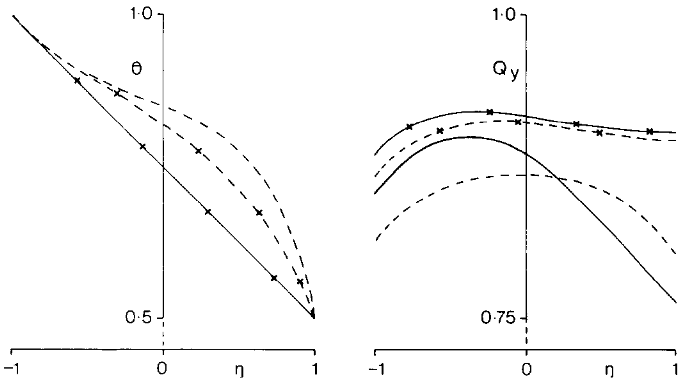
In discussing these results it should be noted from the form of the governing equations themselves that, apart from their influence upon the magnetic field, it is only as a sum that the wall conductivities K_1 and K_2 affect the field variables. This fact remains unchanged whether the absorption coefficient be constant or not. In the cases presented here the electrical and emissive properties of the walls have been taken identical so that $K_1 = K_2 = K$, $\varepsilon_1 = \varepsilon_2 = \varepsilon$. Also as indicated above the parametric values $m = 1$, $n = 5$ have been employed. Further, since the velocity and magnetic field profiles are well established, consideration is restricted to the temperature and radiative flux. It was found that the values of K and M have very little influence upon these and therefore the graphs displayed relate to the case $M = 1$, $K = 0$ corresponding to a moderate electromagnetic interaction in a channel with electrically insulating walls.

The parameter β which provides a measure of the effect of density variations, through thermal expansion, upon the absorption coefficient is singled out in Fig. 1. For increasing β and thus, with temperatures



$\Theta = 0.5$ $\varepsilon = 1$ $K = 0$ $M = 1$ $N = 100$ $F = 0.01$ $\omega = 0.1$
——— $\beta = 0$
— — — $\beta = 10$
------------ $\beta = 100$

FIG. 1. Variable absorption coefficient. Effect of parameter β.



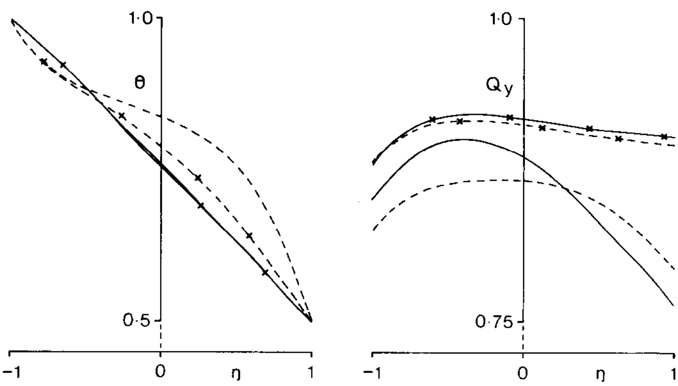
$\Theta = 0.5 \quad \varepsilon = 1 \quad K = 0 \quad M = 1 \quad N = 100 \quad \omega = 0.1 \quad \beta = 0$

———— $F = 0$ —x—x—x—

- - - - - - $F = 0.1$ -x- - -x- - -x- - -

α constant α variable

FIG. 2. Constant and variable absorption coefficient. Effect of parameter F.



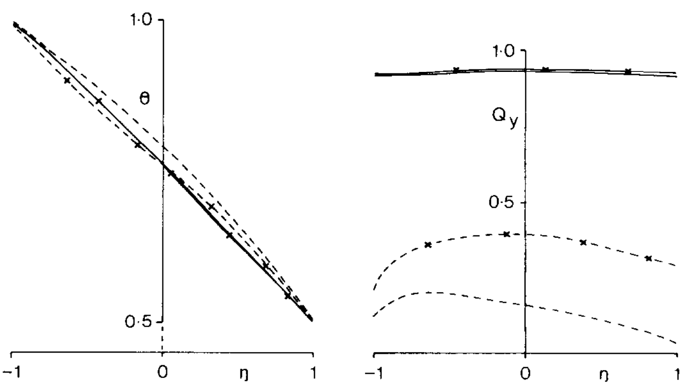
$\Theta = 0.5 \quad \varepsilon = 1 \quad K = 0 \quad M = 1 \quad F = 0.01 \quad \omega = 0.1 \quad \beta = 0$

———— $N = 10$ —x—x—x—

- - - - - - $N = 1000$ -x- - -x- - -x- - -

α constant α variable

FIG. 3. Constant and variable absorption coefficient. Effect of parameter N.



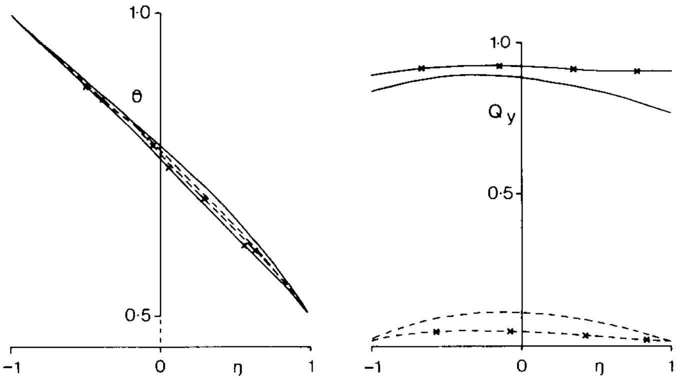
$\Theta = 0.5 \quad \varepsilon = 1 \quad K = 0 \quad M = 1 \quad N = 100 \quad F = 0.01 \quad \beta = 0$

———— $\omega = 0.01$ —x—x—x—

- - - - - - $\omega = 4.0$ -x- - -x- - -x- - -

α constant α variable

FIG. 4. Constant and variable absorption coefficient. Effect of parameter ω.



$\Theta = 0.5 \quad K = 0 \quad M = 1 \quad N = 100 \quad F = 0.01 \quad \omega = 0.1 \quad \beta = 0$

———— $\varepsilon = 1$ —x—x—x—

- - - - - - $\varepsilon = 0.05$ --x----x----x---

α constant $\qquad$ α variable

FIG. 5. Constant and variable absorption coefficient. Effect of parameter ε.

everywhere less than that at datum level, a correspondingly increasing absorption coefficient the variation of the temperature profile becomes a little more marked whilst the magnitude of the radiative flux becomes significantly less.

Figures 2–5 illustrate the effect of variation of the parameters F, N, ω and ε respectively, at the same time now singling out the influence of the fourth power law dependence of the absorption coefficient upon the temperature by choosing $\beta \equiv 0$. The changes in profile consequent upon changes in F, N and ω as measures of the relative importance of thermal conduction, radiative, viscous and fluid convective effects are in agreement with those of earlier workers. The novel information portrayed is that the effect of introducing a variable absorption coefficient whilst quite apparent is not of much significance and qualitative trends in the profiles are unaffected. A perhaps surprising conclusion is to be drawn from Fig. 5 that gross changes in wall emissivity have virtually no effect upon the temperature distribution whilst the local radiative flux itself is, of course, greatly altered.

5. THE EFFECT OF VARIABLE WALL TEMPERATURE AND THERMAL CONVECTION

Consider now the consequences of a non-uniform temperature distribution along the channel walls together with convective effects arising from thermal expansion. Viskanta [11] studied a related problem in a purely gas dynamic context with no electromagnetic effects, whilst Greif *et al.* [10] and Gupta and Gupta [12] have examined the magnetohydrodynamic case but only under the optically thin limit for radiation. The present work extends the model to cover situations without restriction upon the opacity in magnetohydrodynamics.

In Section 4 it has been noted that the introduction into the analysis of a variable absorption coefficient does not have a very significant effect upon the temperature and flux distributions. Thus in order to simplify the problem, hopefully without much loss of generality, the absorption coefficient will now be taken constant. Hence throughout equations (33)–(41) the constants m and n should be set zero. The contribution $Q_x^{(2)}$ to the radiative flux down the channel is then seen, from equation (41) to be identically zero.

In the absence of radiation an exact solution may be obtained by setting the parameter N also equal to zero. The algebraic form is not presented here on account of its complexity, but details may be found in Mosa [19].

When radiative effects are taken into account the full system of equations (32)–(40) are to be solved with boundary conditions (42) and (43). By an appropriate choice of subsidiary variables the system may be reduced to a set of twelve first order nonlinear differential equations and a single algebraic equation. The boundary conditions are split equally between the two ends of the range of integration so that the computational problem is non-trivial. The same general program for the solution as that used in Section 4 is available. An iterative method is employed and in the present instance the nonradiative analytic solution obtained above may be taken as the base from which to develop the iterations.

The outcome of calculations for the velocity, magnetic field and temperature are presented in Figs. (6), (7) and (8) whilst the corresponding radiative flux is portrayed in Fig. (9). Except where otherwise stated on the figures the following values for the parameters are used in these calculations

$$Gr = 1000, \quad F = 0.001, \quad Re = 100,$$
$$N = 100, \quad Ec = 0.001, \quad \gamma = 5/3,$$
$$\omega = 0.1, \quad \varepsilon_1 = \varepsilon_2 = 1, \quad K_1 = K_2 = 0.$$

Although computations have been carried out for several values of the wall temperature ratios, the graphs presented here are typical and correspond to

$$\Theta = 0.5. \quad \Theta_1 = \Theta_2 = 1.$$

Detailed results for other values of the parameters are to be found in the work of Mosa [19]. Particular attention is given to the effect of varying the parameter τ, increasing magnitude of which corresponds to greater variations of wall temperatures; positive values are associated with an increase of temperature downstream, and negative values with a decrease.

Because of the manner in which the various physical parameters for a particular configuration combine to form the non-dimensional parameters and the fact that the latter are specified, and indeed varied independently, rather than the former, it is not possible to lay down precisely the physical dimensions for a particular problem to which all the solutions appertain. Specifically, however, the numerical details relate to gaseous rather than liquid flows. On a broad canvas the results may be expected to apply under conditions as follows:

temperature	10^3–10^4 K,
fluid density	10^{-3}–10^{-5} gm/cm^3,
mean fluid speed	10^3–10^4 cm/s,
channel width	10^{-1}–10 cm,
magnetic field	0–10^4 G (depending on fluid electrical conductivity).

In Fig. 6 the classical influence of Hartmann number upon the velocity profile is seen to be little changed for small wall non-uniformity. However, it is apparent that as this non-uniformity becomes more marked the convective forces cause a distortion of the profile which for even quite modest temperature variation is significant. Similar effects upon the magnetic field are shown in Fig. 7.

The temperature in the channel is given substantially by θ since the additional contribution from $\bar{\theta}$ is always to be multiplied by τ and the computations never yield $\bar{\theta}$ large. For a similar reason the transverse radiative flux is very approximately given by $Q_y^{(1)}$. As radiative energy becomes more substantial, as represented by increasing N, the relatively small changes in radiative flux lead to a considerable flattening of the temperature profile in mid-channel when wall non-uniformity is small.

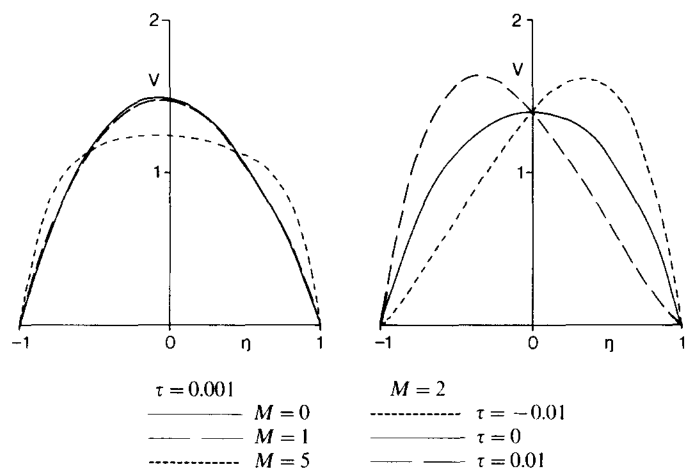


FIG. 6. Velocity profiles.

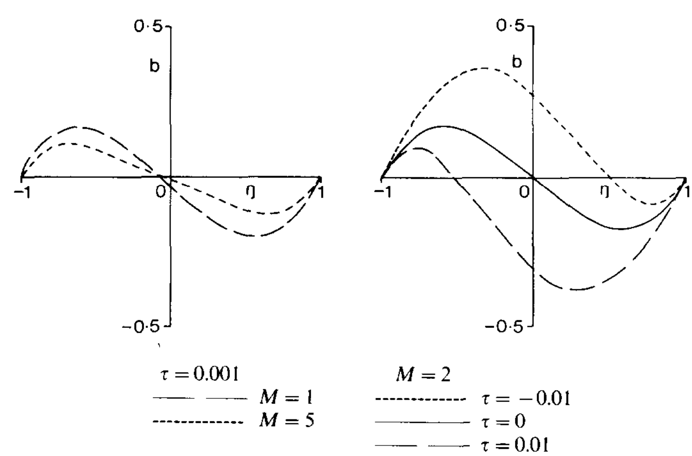


FIG. 7. Magnetic field profiles.

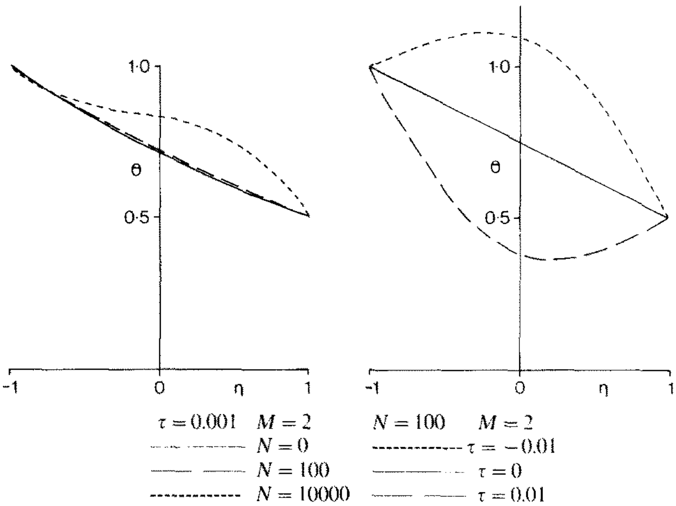


FIG. 8. Temperature profiles.

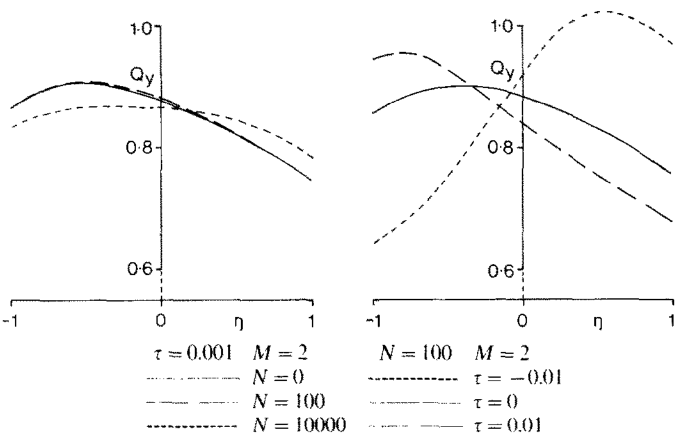


FIG. 9. Transverse radiative flux profiles.

Figures (8) and (9) however indicate that as the non-uniformity becomes more substantial the convective forces oppose this effect; indeed the influence of buoyancy forces upon the temperature and flux is marked and the flattening becomes less apparent. The temperature in mid-channel is considerably lowered as τ increases with a corresponding broad decrease of radiative flux in the upper half channel and enhancement in the lower half.

The streamwise radiative flux is given by $\tau Q_x^{(1)}$. Computation shows that with increase of wall temperature downstream there is an upstream flux of radiation which, despite the considerable variation of temperature across the channel, remains fairly uniform. It is little affected by changes of wall emissivity but is strongly influenced by the fluid optical thickness, as was noted earlier in Section 4 for the transverse component.

6. SUMMARY OF CONCLUSIONS

The paper examines the interaction of conduction, convection and radiation on heat transfer in an electrically conducting fluid confined in a horizontal channel in the presence of a transverse magnetic field. Study is concentrated on two particular aspects.

First a comparison is made between the predicted profiles based upon (a) the commonly used constant absorption coefficient and (b) the more realistic coefficient dependent upon local fluid density and temperature. The detailed conclusions are to be found in Section 4, but for the convenience of the reader the main points are summarised here.

1. Whilst qualitatively unchanged the predicted variation across the channel in the profile of the radiative flux is suppressed by use of a variable absorption coefficient whilst the temperature profile becomes more similar to the linear form associated with pure conduction. The effects are not particularly marked when the temperatures of the walls are of the same order of magnitude.

2. Changes in the coefficient of thermal expansion have a significant effect upon the radiative flux which is reduced in magnitude when the temperature variation leads to a rise in fluid density.

3. The temperature distribution is essentially unaffected by changes in wall emissivity.

Secondly, the influence of a longitudinal variation of wall temperature upon the transverse variation of velocity, temperature and radiative flux is analysed. The problem is studied in Section 5 from which the main conclusions are:

(i) The effect of longitudinal thermal convection has a dominating influence upon the profiles.

(ii) Distortion of the velocity profile is such that an increase of wall temperature downstream leads to a rise of velocity in the lower half of the channel and associated reduction in the upper half. A sufficiently large increase of wall temperature downstream may lead to flow reversal near the upper wall when this is cooler than the lower wall.

(iii) Enhancement of radiative flux from a lower wall heated downstream leads to a more rapid fall of temperature in the lower half of the channel than in the absence of warming; the reverse trend occurs with cooling.

Acknowledgement—One of the authors (M.F.M.) is indebted to the Ministry of Higher Education and Scientific Research of the Government of Iraq for providing a scholarship and financial support to enable him to carry out this research.

REFERENCES

1. R. Siegel, Effect of magnetic field on forced convective heat transfer in a parallel plate channel, *J. Appl. Mech.* **25**, 415 (1958).
2. R. A. Alpher, Heat transfer in magnetohydrodynamic flow between parallel plates, *Int. J. Heat Mass Transfer* **3**, 108 (1961).
3. G. Z. Gershuni and E. M. Zhukhovitsky, Stationary convection flow of an electrically conducting liquid between parallel plates in a magnetic field, *Soviet Phys. JETP* **34**(7), 461 (1958).
4. C. P. Yu, Combined forced and free convection channel flows in magnetohydrodynamics, *AIAA Jl* **3**, 1184 (1965).
5. C. C. Chang and J. T. Yen, Magnetohydrodynamic channel flow as influenced by wall conductance, *Z. Angew. Math. Phys.* **13**, 266 (1962).
6. W. N. Gill and A. D. Casal, A theoretical investigation of natural convection effects in forced horizontal flows, *A.I.Ch.E. Jl* **8**, 513 (1962).
7. A. S. Gupta, Combined free and forced convection effects on the magnetohydrodynamic flow through a channel, *Z. Angew. Math. Phys.* **20**, 506 (1969).
8. R. N. Jana, Wall conductance effects on convective horizontal channel flow, *Z. Angew. Math. Phys.* **26**, 315 (1975).
9. V. M. Soundalgekar, Free convection effects on the oscillatory flow of an incompressible electrically conducting, viscous fluid past an infinite, vertical porous plate with constant suction and transverse magnetic field, *Z. Angew. Math. Mech.* **55**, 257 (1975).
10. R. Greif, I. S. Habib and J. C. Lin, Laminar convection of a radiating gas in a vertical channel, *J. Fluid Mech.* **46**, 513 (1971).
11. R. Viskanta, Effect of transverse magnetic field on heat transfer to an electrically conducting and thermally radiating fluid flowing in a parallel plate channel, *Z. Angew. Math. Phys.* **14**, 353 (1963).
12. P. S. Gupta and A. S. Gupta, Radiation effect on hydromagnetic convection in a vertical channel, *Int. J. Heat Mass Transfer* **17**, 1437 (1974).
13. J. B. Helliwell, Effects of thermal radiation in magnetohydrodynamic channel flow, *J. Engng Math.* **7**, 11 (1973).
14. J. B. Helliwell, Effects of thermal conductivity in radiative magnetohydrodynamic channel flow, *J. Engng Math.* **7**, 347 (1973).
15. B. H. Armstrong, J. Sokoloff, R. W. Nicholls, D. H. Holland and R. E. Meyerott, Radiative properties of high temperature air, *Jl Quantve Spectros. Radiat. Transf.* **1**, 143 (1961).
16. W. G. Vincenti and C. H. Kruger, *Introduction to Physical Gas Dynamics*, Chapters 11 and 12. John Wiley, New York (1965).
17. J. A. Shercliff, *A Textbook of Magnetohydrodynamics*. Pergamon Press, Oxford (1965).
18. R. D. Cess, On the differential approximation in radiative transfer, *Z. Angew. Math. Phys.* **17**, 776 (1966).
19. M. F. Mosa, Ph.D. Dissertation, University of Bradford (in preparation).
20. M. N. Ozisik, *Radiative Transfer and Interactions with Conduction and Convection*, Chapter 12. John Wiley, New York (1973).
21. NAG, Numerical Algorithms Group, NAG Library Manual, Doc 508 (1973).

TRANSFERT THERMIQUE RADIATIF DANS UN ECOULEMENT MAGNETOHYDRODYNAMIQUE A L'INTERIEUR D'UNE CONDUITE HORIZONTALE, AVEC EFFET DE CONVECTION NATURELLE ET GRADIENT AXIAL DE TEMPERATURE

Résumé—Des études de l'écoulement d'un fluide chaud et électriquement conducteur, dans un canal horizontal rectangulaire avec un champ magnétique transversal et un transfert thermique par rayonnement, sont étendues pour prendre en compte ces deux effets. On considère une dépendance en loi puissance du coefficient d'absorption en fonction de la température. Une comparaison avec les résultats d'un coefficient d'absorption constant montre que les conséquences ne sont pas particulièrement marquées. Le second effet considéré est celui de l'influence des forces d'Archimède et du transfert thermique convectif quand les parois du canal sont chauffées différemment et non-uniformément. Dans ce cas il y a une contribution significative aux profils qui sont considérablement distordus par rapport à ceux qui correspondent à des températures de paroi uniformes.

Dans cette étude, le gaz a une opacité générale pour le transfert radiatif et les parois ont une conductivité électrique et une émissivité arbitraire. On prend aussi en compte la conduction thermique moléculaire, la viscosité et la dissipation ohmique. On obtient quelques solutions exactes mais, en général, les équations aux dérivées partielles sont intégrées numériquement et les résultats présentés graphiquement.

WÄRMETRANSPORT DURCH STRAHLUNG BEI HORIZONTALER MAGNETOHYDRODYNAMISCHER KANALSTRÖMUNG MIT AUFTRIEBSEFFEKTEN UND EINEM AXIALEN TEMPERATUR-GRADIENTEN

Zusammenfassung—Es wurden Strömungsuntersuchungen an einem heißen, elektrisch leitenden Fluid in einem horizontalen Rechteck-Kanal in Anwesenheit eines quergerichteten Magnetfeldes und spürbarem Wärmeübergang durch thermische Strahlung durchgeführt, und zwar mit dem Ziel, zwei Effekte besonders zu untersuchen. Zum ersten wurde die Abhängigkeit des Absorptionsgrades von der Temperatur in Form eines Potenzgesetzes betrachtet. Ein Vergleich mit den Ergebnissen für konstanten Absorptionsgrad zeigt, daß dieser Einfluß nicht besonders ausgeprägt ist. Der zweite der erwähnten Effekte ist derjenige, der durch den Einfluß von Auftriebskräften und durch konvektiven Wärmeübergang entsteht, wenn die Kanalwände nur teilweise und ungleichmäßig beheizt sind. In diesem Fall ergibt sich ein wichtiger Einfluß auf die Feldprofile, die erheblich gestört sind gegenüber denen bei gleichförmigen Wandtemperaturen. Bei allen Untersuchungen wurde angenommen, daß das Gas für Strahlung durchlässig ist und daß die Wände beliebige elektrische Leitfähigkeit und beliebigen Emissionsgrad besitzen. Molekulare Wärmeleitung, Viskosität und Ohm'sche Dissipation sind sämtlich in Betracht gezogen worden. Es wurden einige wenige exakte Lösungen gewonnen, im allgemeinen aber wurden die maßgeblichen Differentialgleichungen numerisch integriert und die Ergebnisse grafisch aufgetragen.

ЛУЧИСТЫЙ ПЕРЕНОС ТЕПЛА В ГОРИЗОНТАЛЬНОМ МАГНИТОГИДРОДИНАМИЧЕСКОМ ПОТОКЕ В КАНАЛЕ ПРИ НАЛИЧИИ СИЛ ПЛАВУЧЕСТИ И АКСИАЛЬНОГО ГРАДИЕНТА ТЕМПЕРАТУР

Аннотация — Проведено исследование течения нагретой электропроводной жидкости в прямоугольном горизонтальном канале при наличии поперечного магнитного поля и значительного лучистого теплового потока с учётом двух факторов. Во-первых, учитывалась степенная зависимость коэффициента абсорбции от температуры. Сравнение с результатами, полученными при постоянном значении коэффициента абсорбции, не выявило каких-либо заметных отличий. Во-вторых, учитывалось влияние сил плавучести и конвективного теплообмена при различной и неоднородной температуре стенок канала. В этом случае профили поля значительно отличались от профилей при однородной температуре стенок. Общим для всех экспериментов было допущение о непроницаемости газа для лучистого потока и произвольной электропроводности и излучательной способности стенок. В опытах учитывалась молекулярная теплопроводность, вязкость и диссипация джоулева тепла. Получено несколько точных решений, однако в основном проведено численное интегрирование дифференциальных уравнений и результаты представлены в виде графиков.